Strength of materials or Mechanics of materials

Contents

- Definition of strength of materials
- Basic terms in strength of material

Load
Point load
Distributed load
Tensile force
Compressive force
Axial force
Transverse force

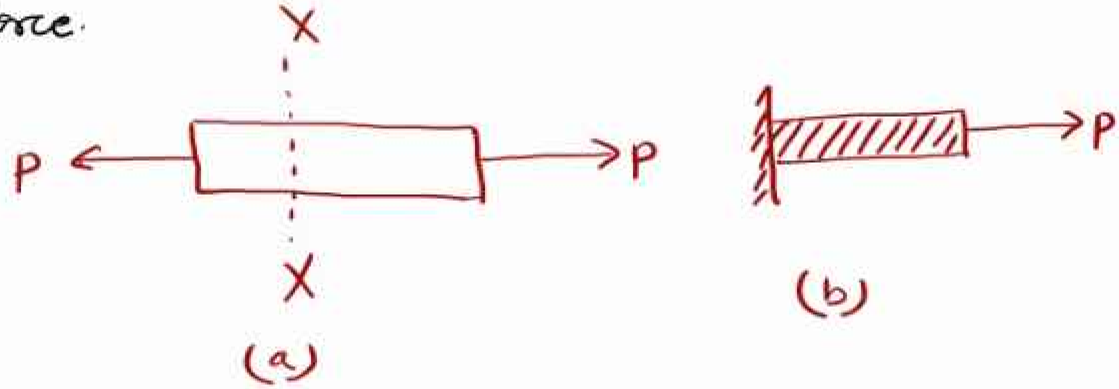
Mechanics of materials :— It is a branch of mechanics that studies the internal effect of stresses and strain in a solid body which is subjected to an external loading.

Load :- The force acting on a body is termed as load.

Point load :- A concentrated load is known as point load.

Distributed load :- Load distribution over a length (or) over an area is known as distributed load.

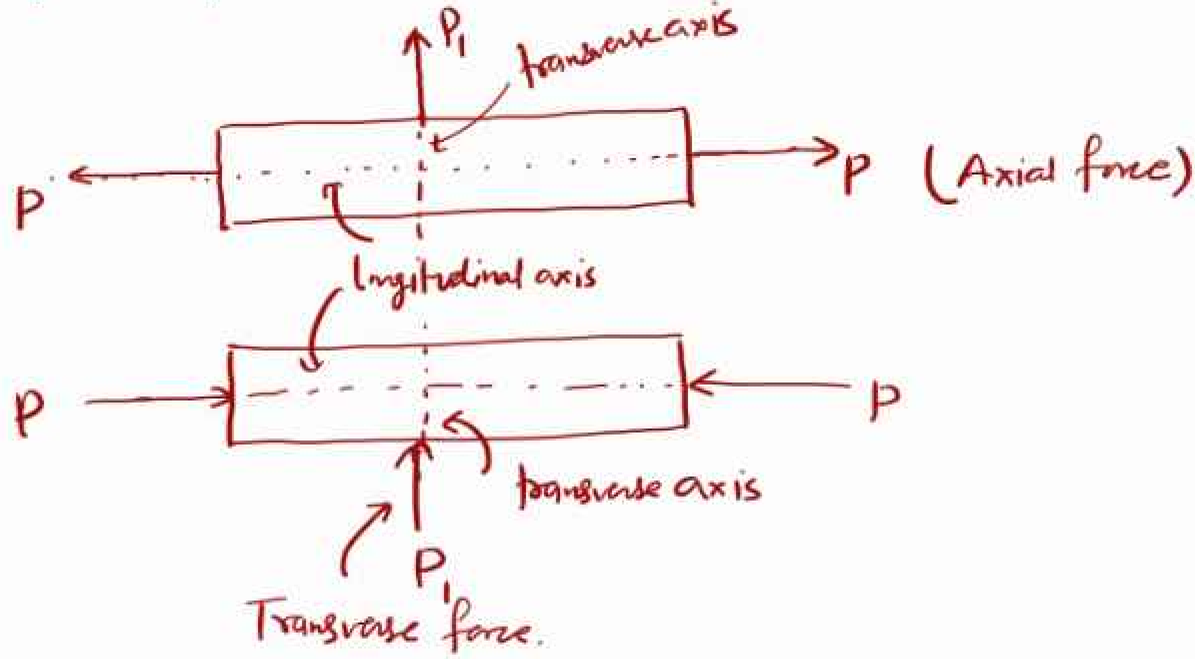
Tensile force :- A load acting on a body which try to pull it apart. Such type of pulling force is known as tensile force.



Compressive force: Force tending to pull (or) compress a body is known as compression (or) compressive force. Such force will tend to shorten the length of member.

Axial force: A force acting on a body along the longitudinal axis are known as axial force.

Transverse force:- Forces acting normal to the longitudinal axis of a body are known as transverse or normal forces.



Strength of materials

Stress AND Strain

Assumptions in strength of material

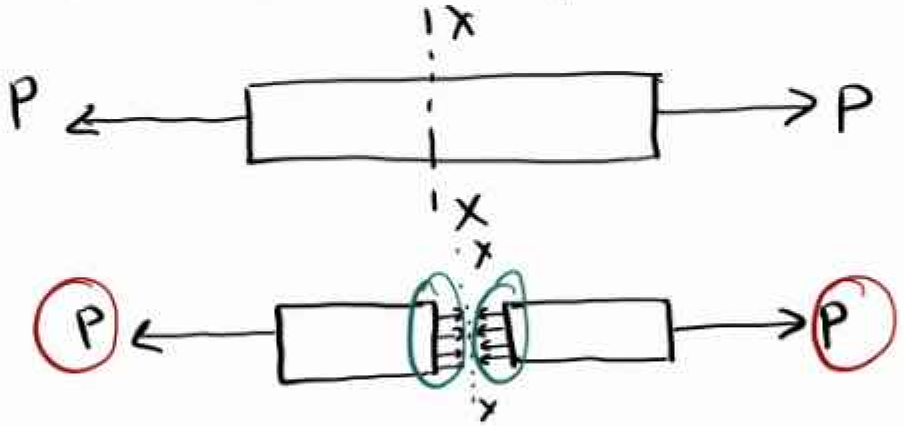
(1) Material subjected to an external forces is assumed to be perfectly elastic.

(2) Materials are isotropic (Same properties in all the directions if a point is considered with a material)

(3) Material is homogeneous (Same properties anywhere within the material)

Stress: (σ) The forces tends to deform the

Solid body and causes it to develop equal and opposite internal forces. These internal forces by virtue of cohesion between particles of the materials tend to resist the deformation.



Now segment of member is in equilibrium under the action of force P and the internal resisting force

The resisting force per unit area of the surface is known as intensity of stress or stress.

So, load = P (Consider this is uniformly distributed over a cross-sectional area)

Cross-sectional area = A

Stress = σ

$$\boxed{\sigma = \frac{P}{A}}$$

If intensity of stress not uniform throughout the body.

$$\sigma = \frac{\delta P}{\delta A}$$

$\delta A \rightarrow$ infinitesimal area of C/s

$\delta P \rightarrow$ load applied on an area δA

Proof stress: The stress at elastic limit.

Unit: N/m^2 (or) Pascal (Pa) (or) MPa (or) GPa

$1\ MPa = 1\ N/mm^2$

$1\ GPa = 1\ kN/mm^2$

Session-3 of Strength of Materials

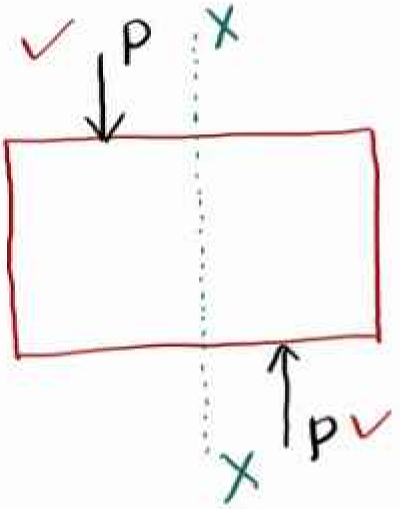
SHEAR STRESS

and

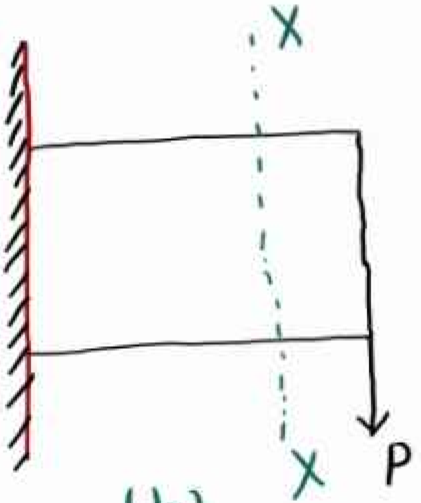
COMPLIMENTORY SHEAR STRSS

Shear Stress

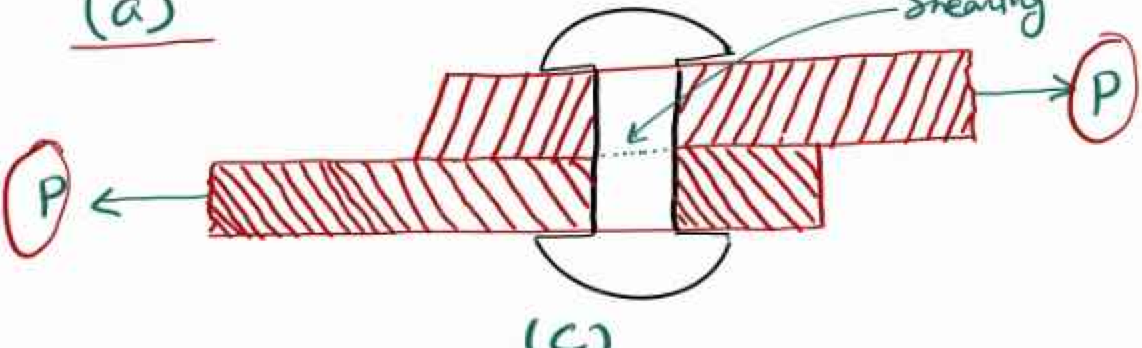
If two equal and opposite parallel forces not in the same line and act on two parts of a body, then one part try to slide over (or) shear from the other across any section and the developed stress is called as shear stress.



(a)



(b)



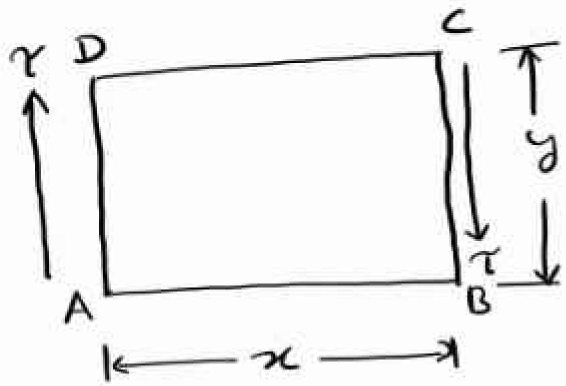
(c)

$$\boxed{\text{Shear Stress } (\tau) = \frac{P}{A}}$$

If intensity of shear stress varies over an area at various points.

$$\boxed{\tau = \frac{\delta P}{\delta A}}$$

Complimentary Shear Stress



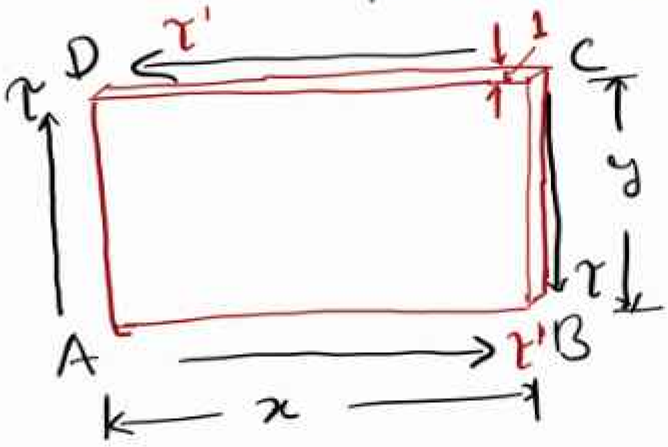
(a)

An elemental area ABCD is considered under shear stress of intensity τ acting on planes AD and BC.

Here, shear stress acting on the element tend to rotate the elemental block in clockwise direction

Only shear stress is acting on the plane AD and BC and other forces are acting on the element but to attain the static equilibrium of element, another couple of the same magnitude is applied in anticlockwise direction.

Here x and y is the length of the plane AB & BC of rectangular element and having unit thickness perpendicular to it.



Force on the couple $= \tau \cdot (y \times 1)$

for face BC

So, Moment of the given couple $=$ (Force on the couple) $\times x$

$= (\tau \cdot y) \cdot x$ —— 1

Similarly,

The force on balancing couple $= \tau' \cdot (x \times 1)$

for face AB

The moment of balancing couple $= (\tau' \cdot x) \cdot y$ —— 2

For Equilibrium condition, Eq^n 1 & 2 must be equal

So, $(\tau \cdot y) \cdot x = (\tau' \cdot x) y$

$\Rightarrow \boxed{\tau = \tau'}$

If a shear stress is applied at the one plane of the body then there must be and equal and opposite shear stress is applied over its perpendicular plane. that is known as complimentary shear stress.

STRAIN

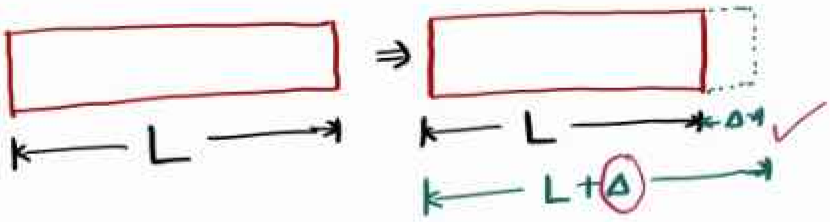
TENSILE STRAIN

COMPRESSIVE STRAIN

SHEAR STRAIN

Strain:

The elongation per unit length in a longitudinal direction of a solid body is known as longitudinal strain (or) simply strain.



If Δ is elongation of a body of length L.

then, $\boxed{\text{Strain} = \dfrac{\text{Change in length}}{\text{Original length}}}$

Symbol = ε

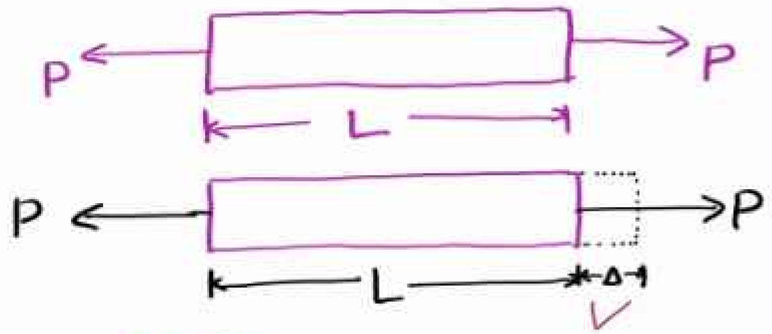
Original length = L, Change in length = Δ

Then, $\boxed{\varepsilon = \dfrac{\Delta}{L}}$

Tensile Strain

A solid member length will increase under axial tensile stress will result increase in length of member from L to $(L+\Delta)$ and Δ is the actual deformation of solid member.

So, tensile strain is ratio of actual deformation Δ to the original length of member.

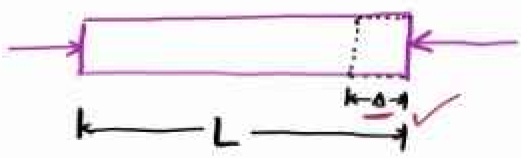


$\boxed{e_t = \dfrac{\Delta}{L}}$

Compressive Strain

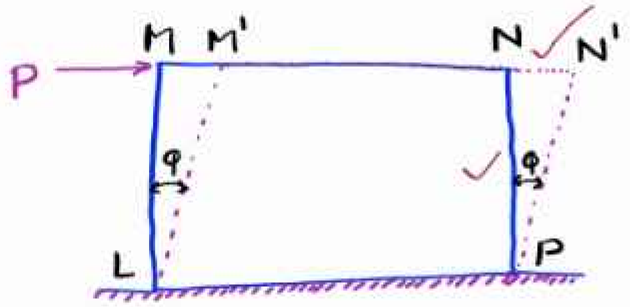
Under application of compressive load, a solid member length will reduce L to $(L-\Delta)$.

So, $\boxed{e_c = \dfrac{\Delta}{L}}$



SHEAR STRAIN

With the application of shearing load, shear strain is produced. It is measured by an angle through which the body distorts.



In a rectangular block LMNP fixed at one face and subjected to force P. Due to application of force, the rectangular block will distort with an angle φ and maintain new position LM'N'P.

The shear strain (e_s) :—

$$e_s = \frac{NN'}{NP} = \tan\varphi \quad \{\varphi \text{ in radian}\}$$

As φ is very small,

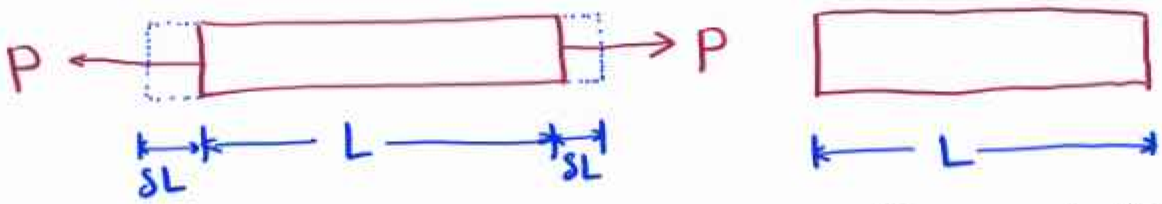
So, $\boxed{e_s = \varphi}$

Mechanics of materials

Elasticity
Plasticity
Hook's Law
Modulus of Elasticity
Modulus of Rigidity
Factor of safety

Elasticity

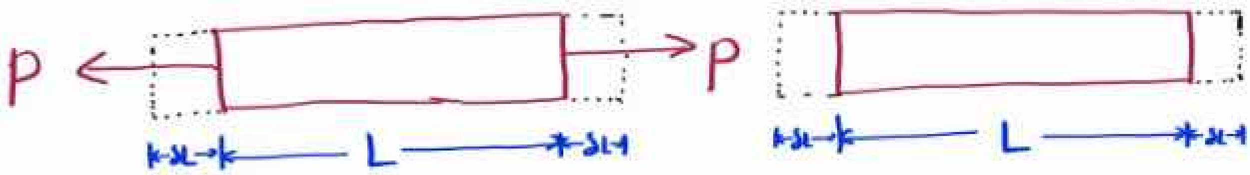
A matertial property by which a material return back to their original shape just after removal of external load is known as Elasticity.



With loading condition After removal of load

Plasticity

Plasticity is a material property in which materials deforms due to the application of load on it but as load removes, material does not recover its deformed shape. This phenomen is called as plasticity



With application of load After removal of load

Hook's law

According to Hook's law, it is Stated that when a material is loaded within elastic limit, the stress is proportional to strain.

Stress $\sigma \propto \varepsilon$ Strain

$\sigma = \dfrac{P}{A}$, $\varepsilon = \dfrac{\Delta}{L}$

(or) $\sigma = E\varepsilon$

Constant (Modulus of Elasticity)

This constant term is known as modulus of Elasticity (or) young's modulus.

Modulus of Elasticity (or) Young's Modulus

The ratio of tensile stress (or) compressive stress to the corresponding strain is constant (E).

$$E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}} \text{ (or) } \frac{\text{Compressive Stress}}{\text{Compressive Strain}}$$

$$\boxed{E = \frac{\sigma}{\varepsilon}}$$

Modulus of Rigidity (or) Shear Modulus

The ratio of shear stress to the corresponding shear strain within the elastic limit.

Symbol = G

$$\boxed{G = \frac{\text{Shear Stress}}{\text{Shear Strain}} = \frac{\tau}{\phi}}$$

Factor of Safety

It is defined as the ratio of tensile stress to the working stress (or) Permissible stress.

$$\boxed{\text{Factor of Safety (FOS)} = \frac{\text{Tensile Stress}}{\text{Working stress}}}$$

Numerical Problem in Mechanics of materials

Stress, # Strain, and # Elongation

A solid member having length of 200 cm long with diameter of 2.5 cm is subjected to an Axial pull force 25 KN. If material having Modulus of Elasticity 2.1×10^5 N/square mm. Evaluate the value of: (E)

1) Stress
2) Strain and, $2.1 \times 10^5\ N/mm^2$
3) Elongation of solid member due to applied pull

Sol^n.

$L = 200\ cm = 200 \times 10\ mm = 2000\ mm$

$\text{diameter} = 2.5\ cm = 2.5 \times 10 = 25\ mm$

$\text{axial force} = 25\ kN = 25000\ N$

$\text{Modulus of elasticity} = 2.1 \times 10^5\ N/mm^2$

① $\text{Stress} = \dfrac{\text{load}}{\text{Area}}$ $\qquad A = \frac{\pi}{4} d^2$

$= \dfrac{25000}{890.87} = 28.06\ N/mm^2 \qquad = \frac{\pi}{4} \times 25^2 = 890.87\ mm^2$

(2) $\text{Strain } (\varepsilon) = \dfrac{\text{Stress}}{\text{modulus of elasticity}}$

$$= \frac{\sigma}{E} = \frac{28.06}{2.1 \times 10^5} = 1.33 \times 10^{-4}$$

$$\varepsilon = \underline{0.000133}$$

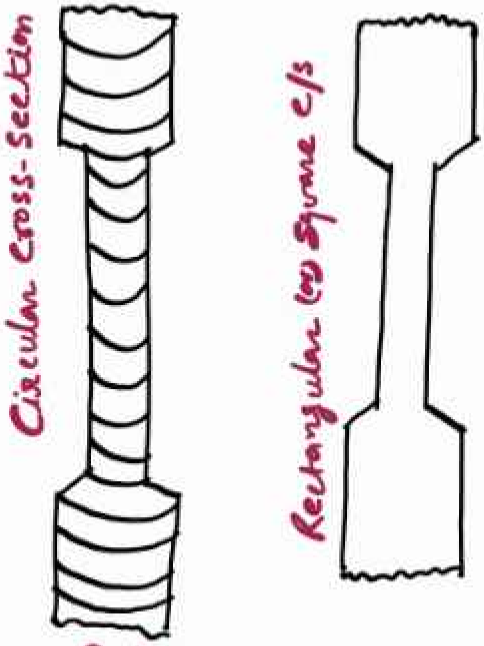
(3) $\varepsilon = \dfrac{\Delta}{L}$

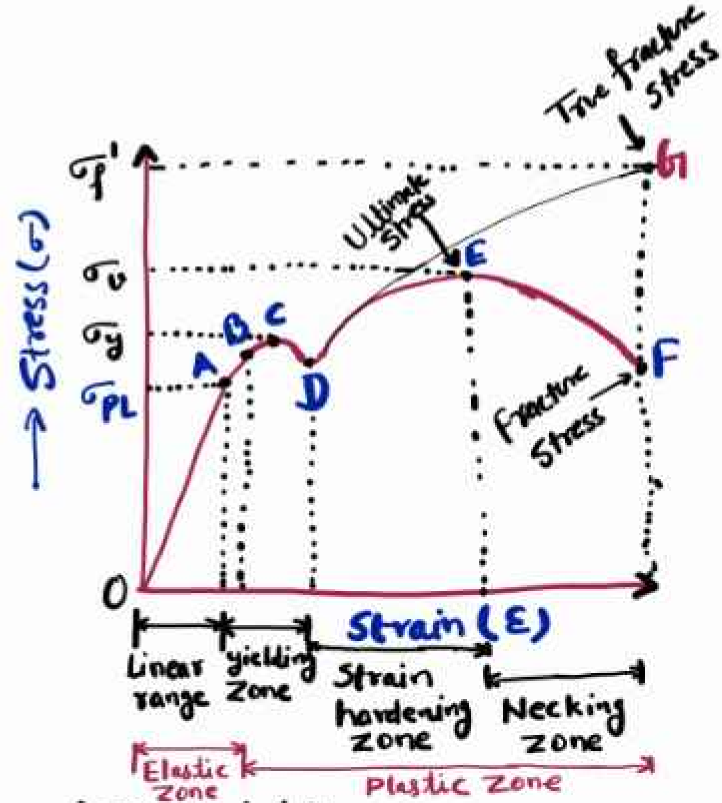
$$\Delta = \varepsilon L = 0.000133 \times 2000 \text{ mm}$$

$$= 0.267 \text{ mm} = 0.0267 \text{ cm}$$

Stress-Strain Diagram for Mild Steel

Mild स्टील के लिये Stress-Strain Curve प्रदर्शित करना सीखें





O = Origin

A = Limit of proportionality

B = Elastic limit

C = Upper yield point

D = lower yield point

AD = yielding zone

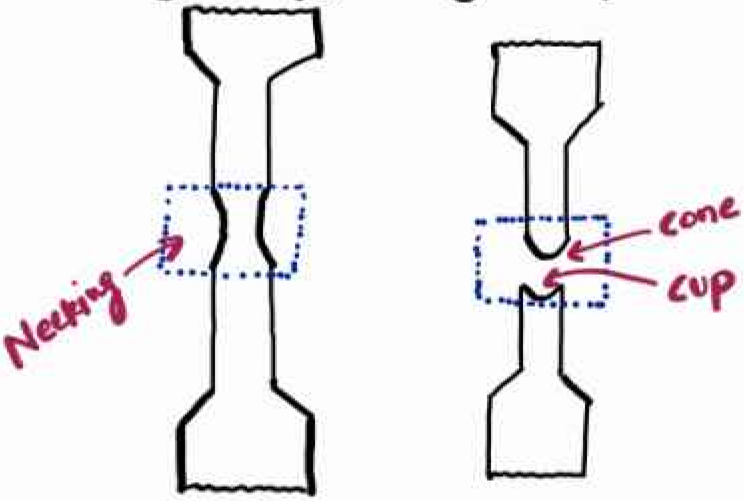
DE = Strain Hardening zone

σ_y = yield stress

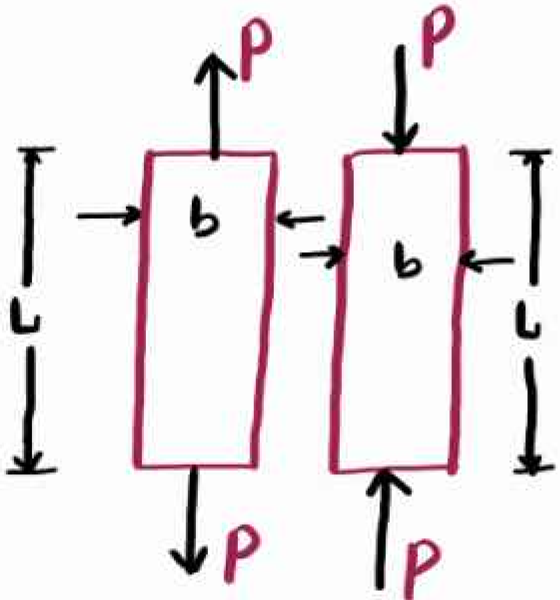
σ_U = Ultimate Stress

F = Failure point

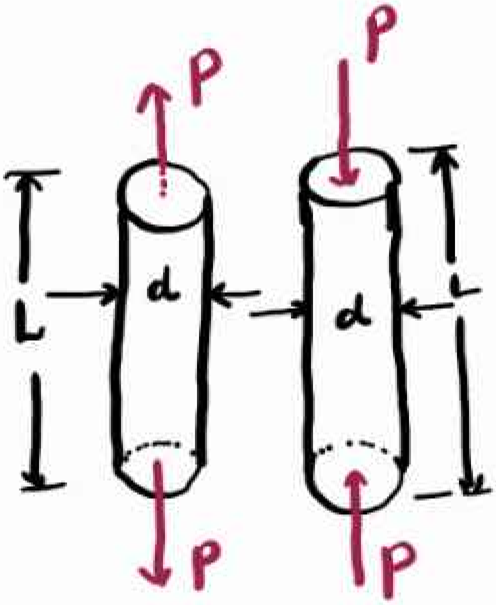
EF = Necking Zone



POISSON'S RATIO (ν)

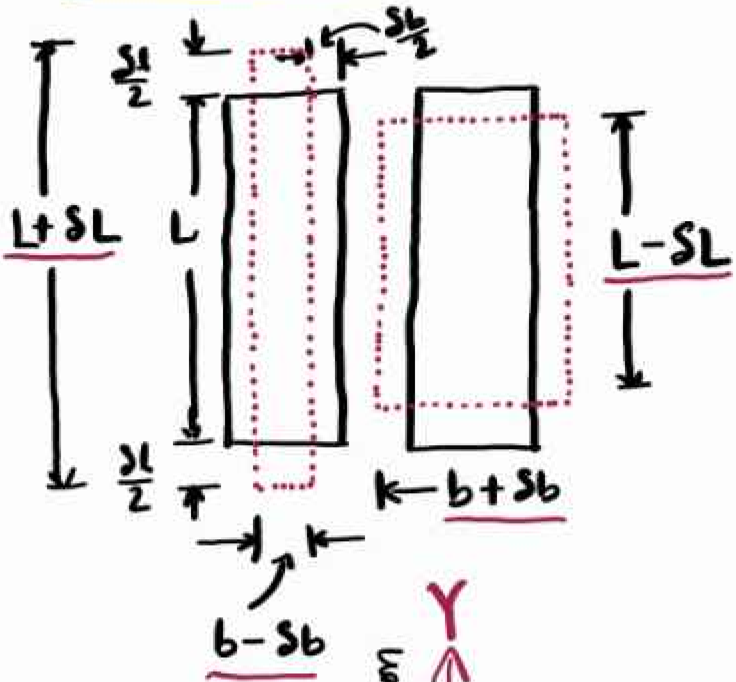


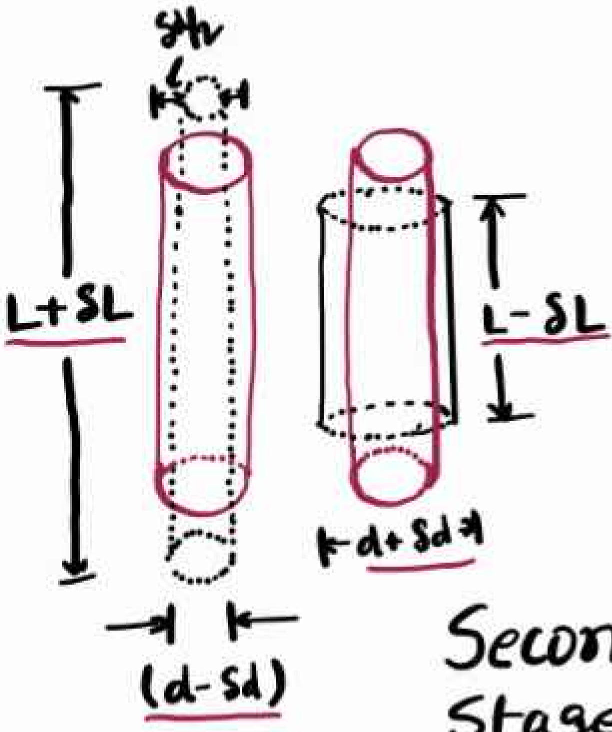
Solid member with Rectangular cross-section



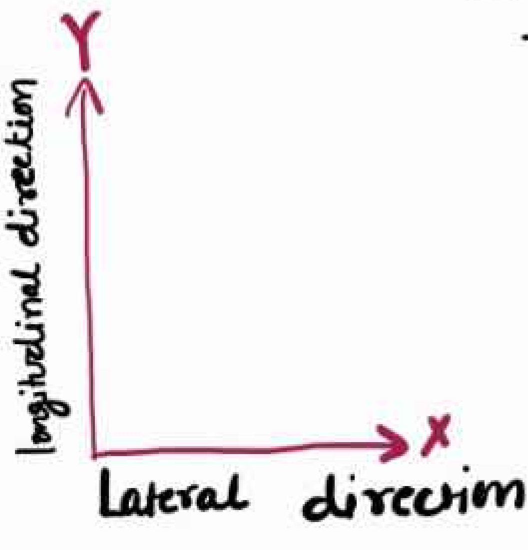
Solid member with Circular cross-section

First Stage





Second Stage



Under tensile force → Dimension in longitudinal direction is increasing
→ Dimension in lateral direction is decreasing

Under Compressive force →
Dimension in longitudinal direction is decreasing
→ Dimension in lateral direction is increasing

Now, Under tension force {(Rectangular cross-section)}

longitudinal strain $= \frac{\delta L}{L}$

lateral strain $= -\frac{\delta b}{b}$

Under compressive force

longitudinal strain $= -\frac{\delta L}{L}$

lateral strain $= \frac{\delta b}{b}$

Circular cross-section

Under tensile force

longitudinal strain $= \frac{\delta L}{L}$

lateral strain $= -\frac{\delta d}{d}$

Under compressive force

longitudinal strain $= -\frac{\delta L}{L}$

lateral strain $= \frac{\delta d}{d}$

Poisson's ratio (ν) $= -\left(\dfrac{\text{Lateral strain}}{\text{longitudinal strain}}\right)$

From above finding

For rectangular cross-section specimen

(i) Under tensile load

$$\nu = \left(\frac{-\frac{\delta b}{b}}{\frac{\delta L}{L}}\right) = -\left(\frac{\text{lateral strain}}{\text{longitudinal strain}}\right)$$

(2) Under compressive load

$$\nu = \left(\frac{\frac{\delta b}{b}}{-\frac{\delta L}{L}}\right) = -\left(\frac{\text{lateral strain}}{\text{longitudinal strain}}\right)$$

For circular cross-section specimen

(i) Under tensile load

$$\nu = \left(\frac{-\frac{\delta d}{d}}{\frac{\delta L}{L}}\right) = -\left(\frac{\text{lateral strain}}{\text{longitudinal strain}}\right)$$

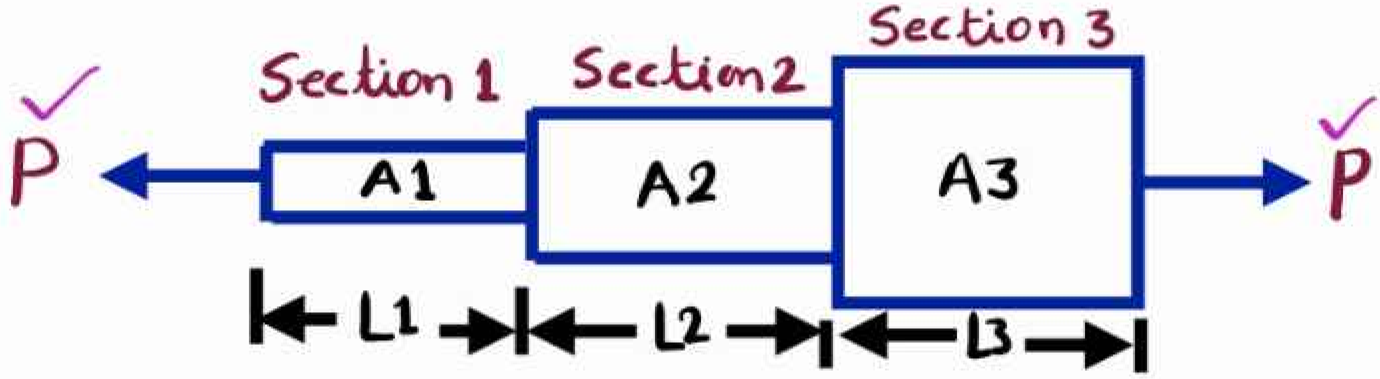
(2) Under compressive load

$$\nu = \left(\frac{\delta d}{d} \Big/ -\frac{\delta L}{L}\right) = -\left(\frac{\text{lateral strain}}{\text{longitudinal strain}}\right)$$

Mechanics of materials

Calculation of material elongation in a solid member with varying sections

Principle of superposition



Axial load $= P$

Length of sections $= L_1, L_2 \,\&\, L_3$

Cross sectional areas $= A_1, A_2 \,\&\, A_3$

Modulus of Elasticity $= E$

$$\left.\begin{aligned} \text{Stress on section 1} &= \frac{P}{A_1} = \sigma_1 \\ \text{Stress on section 2} &= \frac{P}{A_2} = \sigma_2 \\ \text{and Stress on section 3} &= \frac{P}{A_3} = \sigma_3 \end{aligned}\right\} \text{— (1)}$$

$$\left.\begin{aligned} \text{Strain at section 1} &= \frac{\sigma_1}{E} = \varepsilon_1 \\ \text{Strain at section 2} &= \frac{\sigma_2}{E} = \varepsilon_2 \\ \text{and Strain at section 3} &= \frac{\sigma_3}{E} = \varepsilon_3 \end{aligned}\right\} \text{— (2)}$$

$$\left[\begin{aligned} \sigma \propto \varepsilon \Rightarrow \sigma &= E\varepsilon \\ \varepsilon &= \frac{\sigma}{E} \end{aligned}\right]$$

But strain also defines the ratio of change in member length to the original member length.

$$\left.\begin{aligned} \text{Strain at section 1} &= \frac{\Delta_1}{L_1} = \varepsilon_1 \\ \text{Strain at section 2} &= \frac{\Delta_2}{L_2} = \varepsilon_2 \\ \text{and Strain at section 3} &= \frac{\Delta_3}{L_3} = \varepsilon_3 \end{aligned}\right\} \text{— (3)}$$

From formula (2) & (3)

$$\left\{\frac{\sigma}{E}\right\} = \left\{\frac{\Delta}{L}\right\} \quad \left\{\sigma = \frac{P}{A}\right\}$$

$$\frac{P}{AE} = \frac{\Delta}{L}$$

$$\boxed{\Delta = \frac{PL}{AE}}$$

For Section 1

$$\boxed{\Delta_1 = \frac{PL_1}{A_1E}}$$

For section 2

$$\boxed{\Delta_2 = \frac{PL_2}{A_2E}}$$

For section 3

$$\boxed{\Delta_3 = \frac{PL_3}{A_3E}}$$

Stress, Strain & Elongation are determined for Individual Sections.

Now apply Principle of Superposition

If a member experienced number of loads on various segment of a member, then the net effect of loads on the member is the sum of the effect caused by each of the loads acting independently on the respective segment of the member.

So, Total elongation $(\Delta) = \Delta_1 + \Delta_2 + \Delta_3$

$$\Delta = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E}$$

$$\Delta = \frac{P}{E}\left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right] \quad \text{——— } (4)$$

For single material E is same

$$\Delta = P\left[\frac{L_1}{A_1E_1} + \frac{L_2}{A_2E_1} + \frac{L_3}{A_3E_3}\right]$$

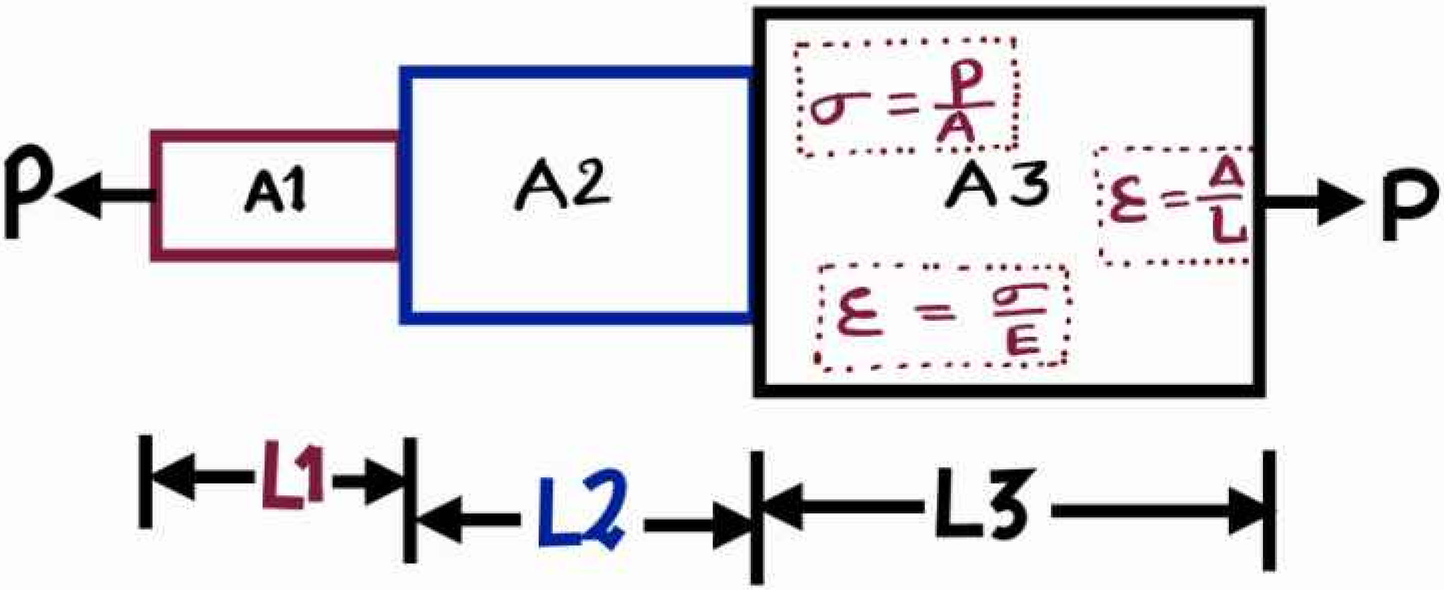
For various material E is different.

Mechanics of materials

Varying cross -section

Principle of superposition

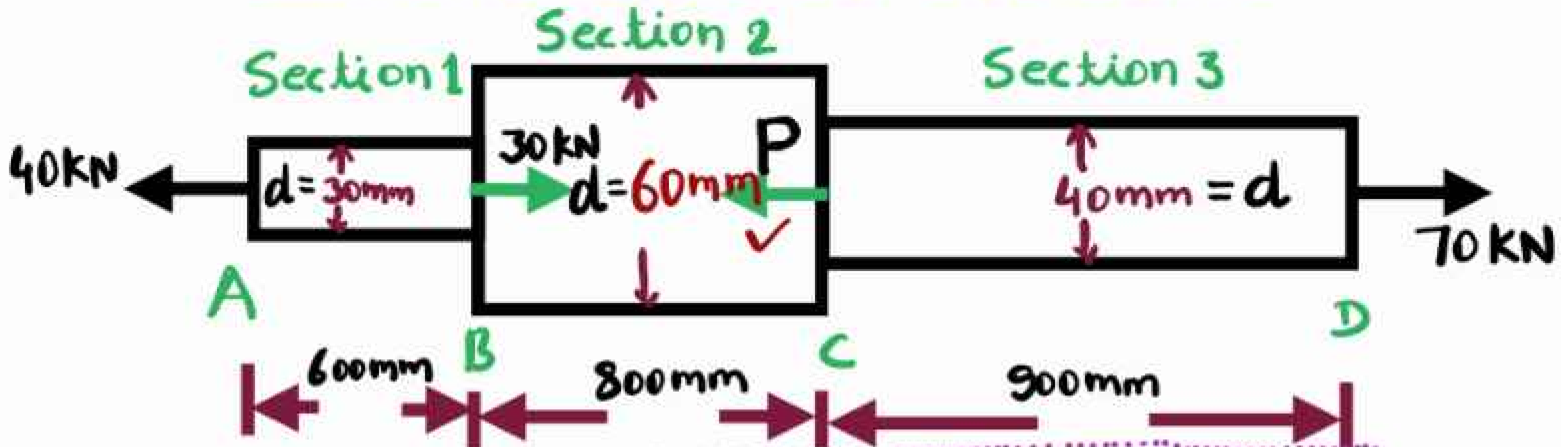
Elongation in varying cross- section



Principle of superposition in Mechanics of Materials

Varying Cross-Section

Determination of forces on different cross-sections



Circular Section

Calculation of loads on section 1, 2 + 3

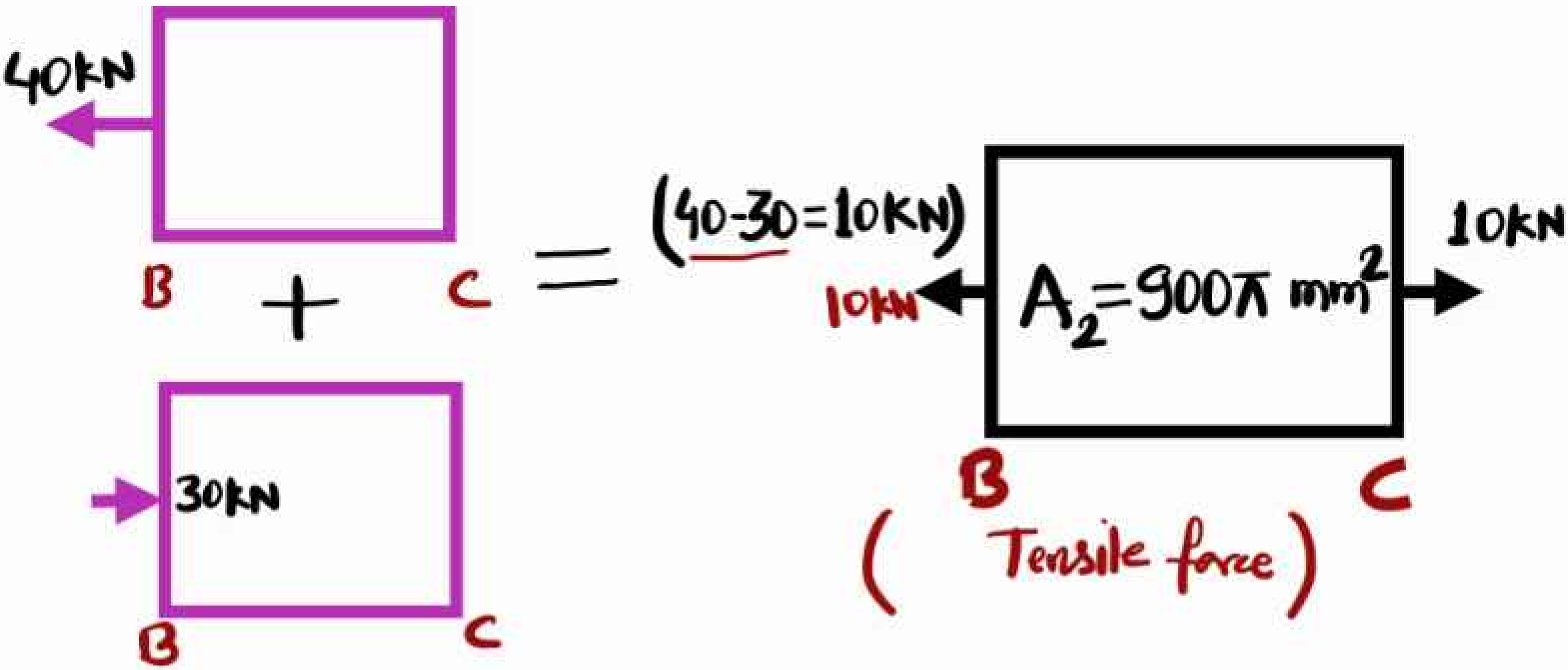
Apply Principle of Superposition

Section 1

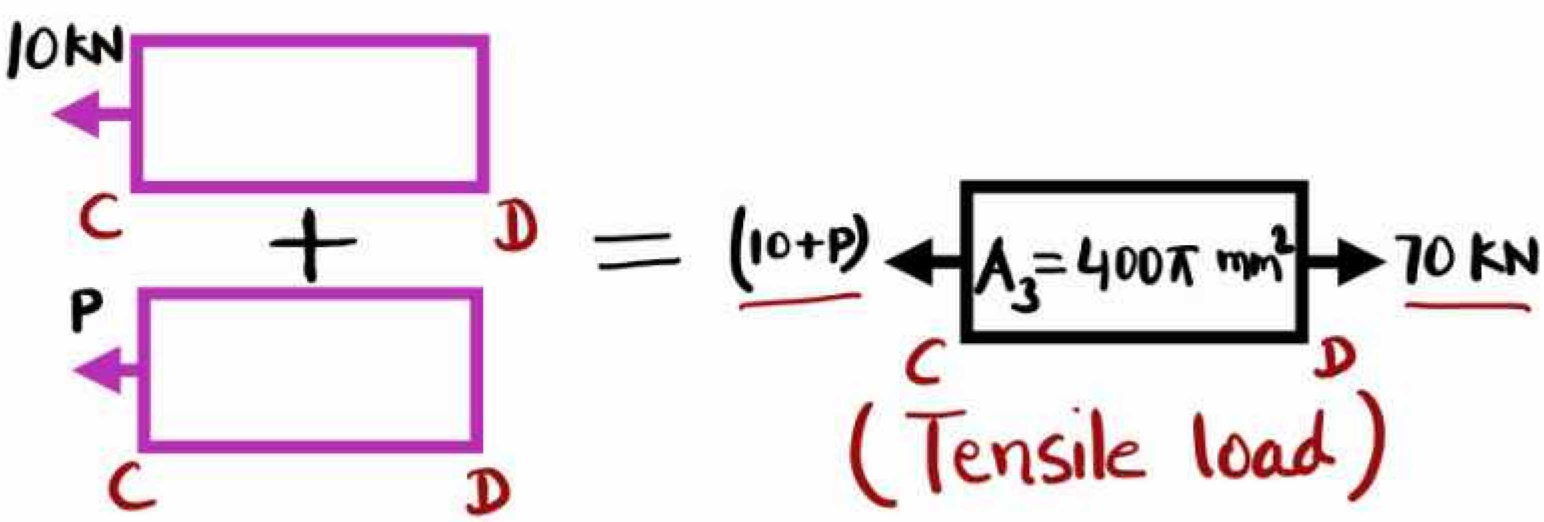
(Tensile load)

left of the section 40KN load is applied. To maintain the section equilibrium, same load applied on right side.

Section 2 (load at face B)



Section 3 (load at face C)



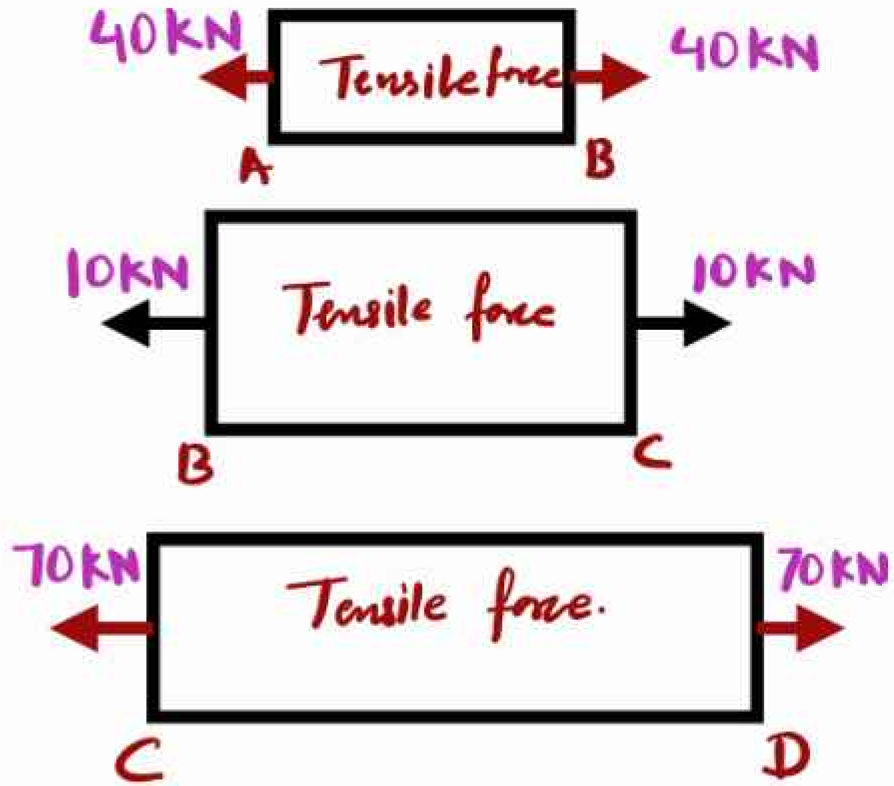
So, $10+P = 70 \Rightarrow P = 70-10 = \boxed{60\,\text{KN}}$

Forces on sections

Section 1

Section 2

Section 3



All sections are made up with same material.

Apply Principle of superposition

Total elongation $= \Delta = \frac{1}{E}\left(\frac{PL}{A}\right) = \Delta_1 + \Delta_2 + \Delta_3$

$$\Delta = \frac{1}{E}\left(\frac{P_1 L_1}{A_1} + \frac{P_2 L_2}{A_2} + \frac{P_3 L_3}{A_3}\right)$$

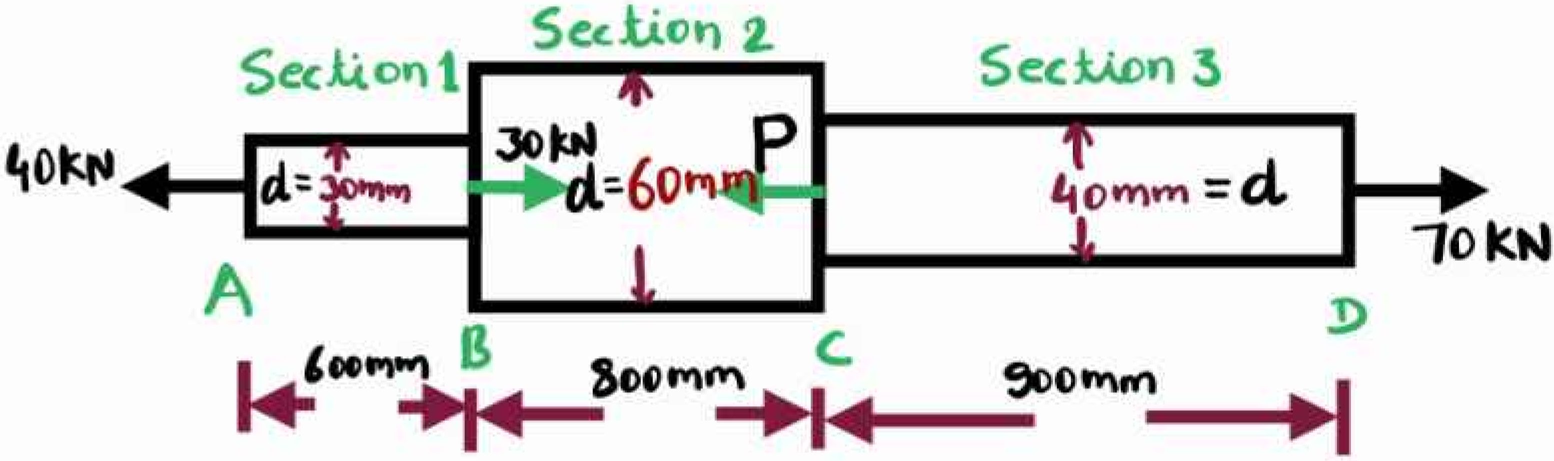
$$= \frac{1}{E}\left(\frac{40000 \times 600}{225\pi} + \frac{10000 \times 800}{900\pi} + \frac{70000 \times 900}{400\pi}\right)$$

$\Delta = \frac{1}{E}(86916.28)$ ..... If $E = 2.1 \times 10^5$ N/mm^2

So, Total elongation $= \Delta = \frac{86916.28}{2.1 \times 10^5} = 0.414$ mm

$\simeq 0.414$ mm

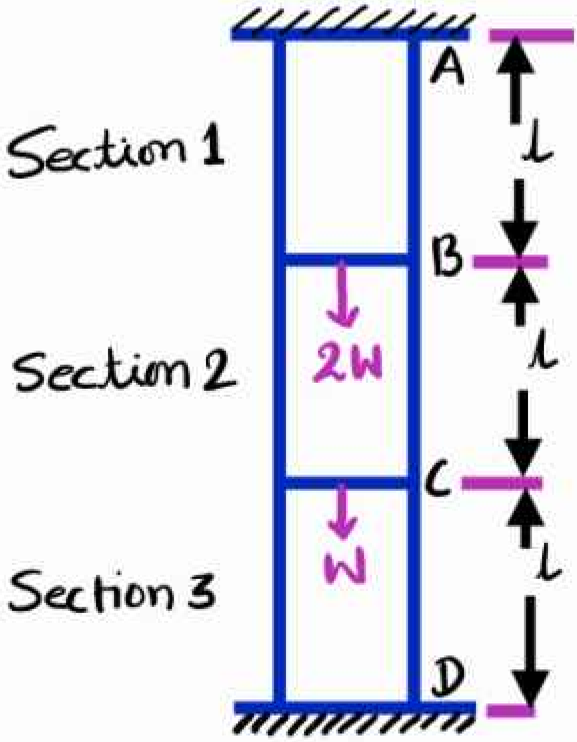
Determin the loads and it's nature on the sections



Apply Principle of superposition

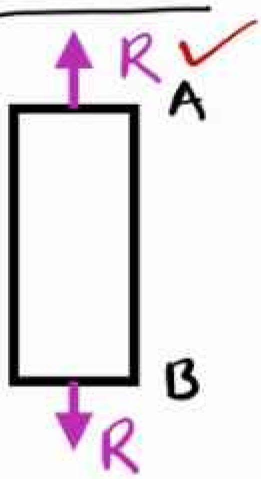
What is total elongation in Solid member

A vertical circular steel of length 3L fixed at both of its ends is loaded at intermediate sections shown in fig.

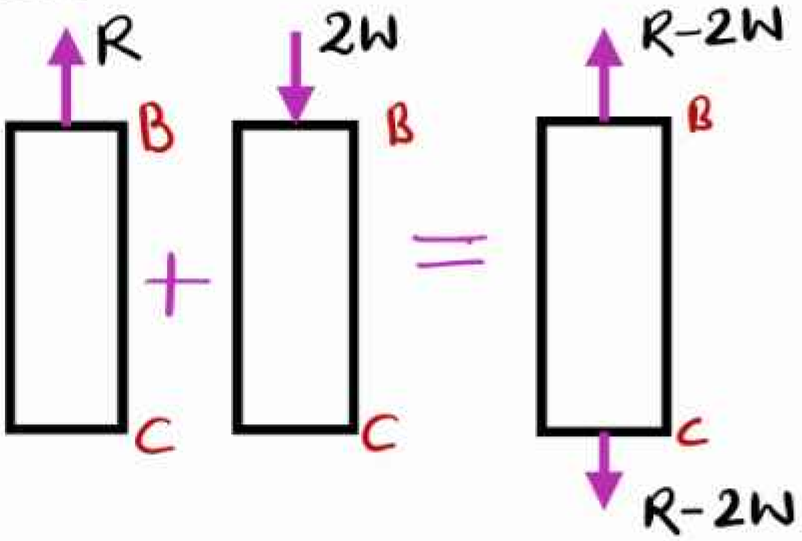


Suppose at point A in section 1, a reaction R is applied.

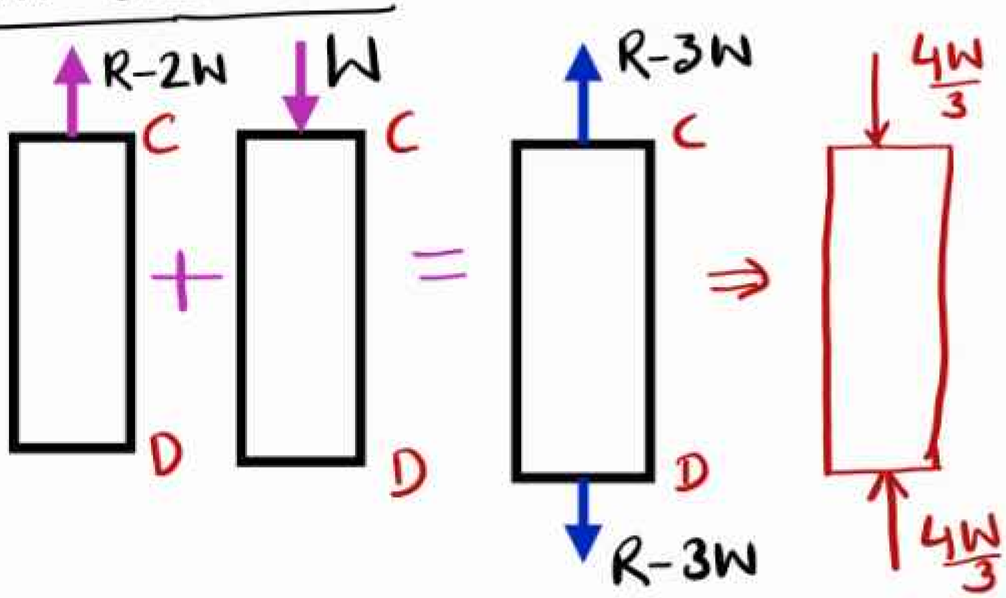
So, For section 1



For section 2



For section 3



As both end is fixed, the elongation is zero.

$$\Delta_1 + \Delta_2 + \Delta_3 = 0$$

$$\frac{RL}{AE} + \frac{(R-2W)L}{AE} + \frac{(R-3W)L}{AE} = 0$$

$$R + R - 2W + R - 3W = 0$$

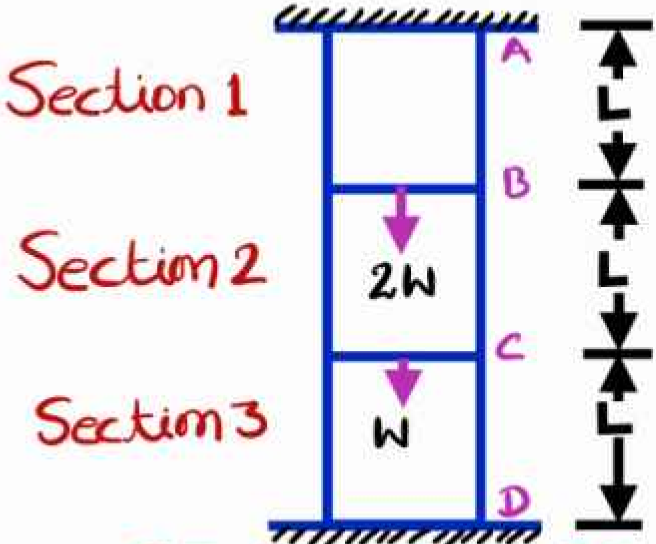
$$3R = 5W \Rightarrow \boxed{R = \frac{5W}{3}}$$ (Reaction at A)

Reaction at lower end $= \underline{R} - 3W$

$= \frac{5W}{3} - 3W = -\frac{4W}{3}$

(Reaction will be in opposite direction) $= -\frac{4W}{3}$

Calculation of member foces



Application of Principle of Superposition